5 SUSY QCD Part I

5.1 Symmetries and Group Theory

F chiral supermultiplets (named after the fermion)

$$Q_i = (\phi_i, Q_i, F_i), i = 1, \ldots, F \tag{5.1}$$
$$\tag{5.2}$$

in the $\mathbf{N}$ of $SU(N)$ and F chiral supermultiplets

$$\overline{Q}_i = (\overline{\phi_i}, \overline{Q_i}, \overline{F_i}) \tag{5.3}$$

in the $\overline{\mathbf{N}}$. Note $^{\overline{\ \ }}$ is part of the name not conjugation

$$Q_i^\dagger = (\phi_i^*, Q_i^\dagger, F_i^*) \tag{5.4}$$
$$\overline{Q}_i^\dagger = (\overline{\phi}_i^*, \overline{Q}_i^\dagger, \overline{F_i^*}) \tag{5.5}$$

The matter content can be summarized as:

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Q	$\Box$	$\Box$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{Q}$	$\overline{\Box}$	$\mathbf{1}$	$\overline{\Box}$	-1	$\frac{F-N}{F}$

The superpotential is $W = 0$. Since the R charge doesn't commute with the SUSY generator:

$$[R, Q_\alpha] = -Q_\alpha, \tag{5.6}$$

we have $R_\psi = R_\phi - 1$, $R\theta = \theta$, and $R\lambda^a = \lambda^a$. Chiral supermultiplets are labeled by the R charge of the scalar component.

Recall the following Group Theory results:

$$(T_R^a)_l^m (T_R^a)_n^l = C_2(R)\delta_n^m \tag{5.7}$$
$$(T_R^a)_n^m (T_R^b)_m^n = T(R)\delta^{ab} \tag{5.8}$$
$$d(R)C_2(R) = d(\mathrm{Ad})T(R) \tag{5.9}$$
$$T(\Box) = \frac{1}{2},\ T(\mathrm{Ad}) = N \tag{5.10}$$
$$C_2(\Box) = \frac{N^2-1}{2N},\ C_2(\mathrm{Ad}) = N \tag{5.11}$$

for the fundamental representation $\mathbf{N}$:

$$(T^a)^l_p (T^a)^m_n = \frac{1}{2}(\delta^l_n \delta^m_p - \frac{1}{N}\delta^l_p \delta^m_n) \tag{5.12}$$

Since we can take linear combinations of $U(1)_R$ and $U(1)$ we can choose Q_i and $\overline{Q}_i$ to have the same R charge. For the $U(1)$ not be to broken by instantons we must have:

$$T(\mathrm{Ad}) + (R-1)T(\Box)2F = 0 \tag{5.13}$$

so

$$R = \frac{F-N}{F} \tag{5.14}$$

5.2 Renormalization Group

At tree-level we saw that the gauge coupling and the quark-squark-gluino Yukawa coupling were related: $Y = \sqrt{2}g$. What happens when the couplings run?

The β function for the gauge coupling is

$$\begin{aligned} \beta_g &= \mu\frac{dg}{d\mu} = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}T(\mathrm{Ad}) - \frac{2}{3}T(F) - \frac{1}{3}T(S)\right) \\ &= -\frac{g^3}{16\pi^2}(3N - F) \end{aligned} \tag{5.15}$$

In the notation of Machacek and Vaughn [2] the Yukawa β function in a general renormalizable gauge theory is given by:

$$\begin{aligned} (4\pi)^2\beta^a_Y &= \frac{1}{2}\left[Y_2^\dagger(F)Y^a + Y^a Y_2(F)\right] + 2Y^b Y^{\dagger a} Y^b \\ &\quad + Y^b \,\mathrm{Tr}\, Y^{\dagger b} Y^a - 3g_m^2\{C_2^m(F), Y^a\} \end{aligned} \tag{5.16}$$

where Y^a_{ij} is the Yukawa coupling of real scalar a to fermions i and j,

$$Y_2(F) = Y^{\dagger a} Y^a \;, \tag{5.17}$$

and $C_2^m(F)$ is the quadratic Casimir of the fermion fields transforming under the m-th gauge group. Thus the first term in Eq. (5.16) represents scalar loop corrections to the fermion legs, the second term 1PI corrections from the Yukawa interactions themselves, the third term fermion loop corrections to the scalar leg, and the last term represents gauge loop corrections to the fermion legs.

$$\begin{aligned}(4\pi)^2\beta_Y^a &= \sqrt{2}g^3(C_2(\Box)+F+2C_2(\Box)-3C_2(\Box)-3N) \\ &= -\sqrt{2}g^3(3N-F) \\ &= \sqrt{2}(4\pi)^2\beta_g \end{aligned} \tag{5.18}$$

There is also a relation between the gauge coupling and the D term quartic coupling $\lambda = g^2$.

$$D^a = g(\phi^{*in}(T^a)_n^m\phi_{mi} - \overline{\phi}^{in}(T^a)_n^m\overline{\phi}^*_{mi}) \tag{5.19}$$

and the potential is:

$$V = \frac{1}{2}D^aD^a \ . \tag{5.20}$$

The β function for a quartic scalar coupling at one-loop in a general renormalizable field theory is given by [2]:

$$(4\pi)^2\beta_\lambda = \Lambda^2 - 4H + 3A + \Lambda^Y - 3\Lambda^S, \tag{5.21}$$

where Λ^2 corresponds to the 1PI contribution from the quartic interactions themselves (not to be confused with a mass scale), H corresponds to the fermion box graphs, A to the two gauge boson exchange graphs, Λ^Y to the Yukawa leg corrections, and finally Λ^S corresponds to the gauge leg corrections. Note that individual diagrams renormalize the gauge invariant (SUSY breaking) operator: $(\phi^{*ni}\phi_{ni})^2$. However the β function for this operator vanishes and the D term β function satisfies

$$\beta_{g^2} = 2g\beta_g \tag{5.22}$$

So SUSY β functions are not anomalous at one-loop, and the relations between couplings are preserved at all scales.

5.3 Quadratic Mass Divergence of the Squark

Calculate the one-loop self-energy corrections for the squark:

Squark loop:

$$\begin{aligned} & -ig^2(T^a)_n^l(T^a)_l^m\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2} \\ &= \frac{-ig^2}{16\pi^2}C_2(\Box)\delta_n^m\int_0^{\Lambda^2} dk^2 \end{aligned} \tag{5.23}$$

Figure 1: Squark loop

Figure 2: Quark-Gluino loop

Quark-gluino loop:

$$
\begin{aligned}
(-i\sqrt{2}g)^2 (T^a)^l_n (T^a)^m_l (-1) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr} P_L \frac{i}{\not{k}} P_R \frac{i}{\not{k}} \\
= \frac{4ig^2}{16\pi^2} C_2(\Box) \delta^m_n \int_0^{\Lambda^2} dk^2
\end{aligned}
\tag{5.24}
$$

Squark-Gluon loop:

Figure 3: Squark-Gluon loop

$$
\begin{aligned}
(ig)^2 (T^a)^l_n (T^a)^m_l \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} k^\mu (-i) \frac{(g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2})}{k^2} k^\nu \\
= \frac{\xi i g^2}{16\pi^2} C_2(\Box) \delta^m_n \int_0^{\Lambda^2} dk^2
\end{aligned}
\tag{5.25}
$$

Figure 4: Gluon loop

Gluon loop:

$$\frac{1}{2} i g^2 \left\{ (T^a)^l_n, (T^b)^m_l \right\} \delta^{ab} g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} (-i) \frac{(g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2})}{k^2}$$
$$= \frac{-(3+\xi) i g^2}{16\pi^2} C_2(\Box) \delta^m_n \int_0^{\Lambda^2} dk^2 \quad (5.26)$$

Thus the quadratic divergence cancels!

References

[1] "Lectures on supersymmetric gauge theories and electric-magnetic duality," by K. Intriligator and N. Seiberg, hep-th/9509066.

[2] M.E. Machacek and M.T. Vaughn, "Two Loop Renormalization Group Equations In A General Quantum Field Theory (I). Wave Function Renormalization," Nucl. Phys. **B222**, 83 (1983). M.E. Machacek and M.T. Vaughn, "Two Loop Renormalization Group Equations In A General Quantum Field Theory (II). Yukawa Couplings," Nucl. Phys. **B236**, 221 (1984). M.E. Machacek and M.T. Vaughn, "Two Loop Renormalization Group Equations In A General Quantum Field Theory (III). Scalar Quartic Couplings," Nucl. Phys. **B249**, 70 (1985).

[3] P. Cvitanovic, "Group Theory For Feynman Diagrams In Non-Abelian Gauge Theories," Phys. Rev. **D14**, 1536 (1976).